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Note

The Calculation of the Pressure in Unsteady Flows Using a Control-Volume Approach

A new method is proposed to calculate the pressure correction in unsteady flows using the finite-difference scheme of the TEACH-T code [1]. The method was originally developed to study the flow field in axisymmetric internal combustion engine configurations [2]; however, its extension to other geometries and three dimensions is obvious. In many flow fields, pressure variations are produced by the compression and expansion of the gas, as in an internal combustion engine; however, pressure variations are also produced by the fluid motions.Since the pressure drives the flow, we must accurately predict the small-scale pressure variations in order to resolve the flow field. To calculate the pressure, we propose to correct the pressure field after the solution of the momentum equations has been obtained in two ways: first, a uniform global pressure correction, denoted by $\overline{p'}$, is made equally to the entire flow field, and second, local pressure corrections, denoted by p', are made which vary from point to point.

The Uniform Global Pressure and Temperature Corrections

In unsteady, axisymmetric flows the continuity equation reads

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v) + \frac{\partial}{\partial z}(\rho u) = 0, \tag{1}$$

where t, r, z, u, v and ρ stand for the time, radial and axial coordinates, axial and radial velocities, and density, respectively. When (1) is integrated over a control volume, as shown in Fig. 1, the expression

$$M_P^{n+1} - M_P^n + \dot{m}_e^{n+1} - \dot{m}_w^{n+1} + \dot{m}_n^{n+1} - \dot{m}_s^{n+1} = 0 \tag{2}$$

is obtained, where

$$M_P^{n+1} = \rho_P^{n+1}\left(\frac{Vol}{\Delta t}\right)_P^{n+1}. \tag{3}$$

$\dot{m}_i$ are the mass fluxes through the i-boundaries of the control volume centered at P;

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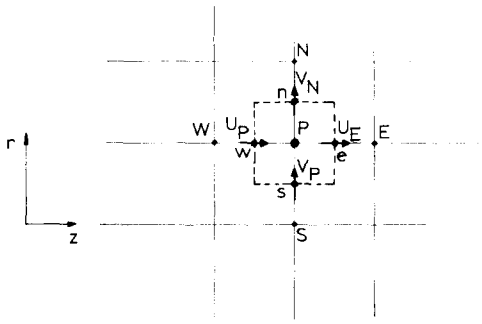

FIG. 1. A computational cell.

Vol is the volume of the cell per radian and Δt is the time-step. Equation (2) can be written as

$$M_P^{n+1} - M_P^n + \dot{m}_c^{n+1} = 0, \tag{4}$$

where

$$\dot{m}_c^{n+1} = \dot{m}_e^{n+1} - \dot{m}_w^{n+1} + \dot{m}_n^{n+1} - \dot{m}_s^{n+1}. \tag{5}$$

When the flow field has converged, (2) is identically satisfied, but suppose that we are iterating within the time-step from t to $t+\Delta t$, and the flow field has not yet converged. At that iteration (*) we have

$$M_P^* - M_P^n + \dot{m}_c^* = 0. \tag{6}$$

Using the equation of state, $\rho = \rho(p, T)$ and assuming that the flow field at the iteration (*) is nearly converged, we can expand ρ in a Taylor series expression as

$$\rho_P^{n+1} = \rho_P^* + \left(\frac{\partial \rho}{\partial p}\right)_T^* \overline{p'} + \left(\frac{\partial \rho}{\partial T}\right)_p^* \overline{T'}, \tag{7}$$

where $\overline{p'}$ and $\overline{T'}$ are the global pressure and temperature corrections. Substituting (7) into (4), and adding the result over the whole computational domain (F), we have

$$\sum_F (M_P^* - M_P^n) + \sum_F \dot{m}_c^{n+1} + \overline{p'} \sum_F \beta_P^* + \overline{T'} \sum_F \gamma_P^* = 0, \tag{8}$$

where

$$\beta_P^* = \left(\frac{\partial \rho}{\partial p}\right)_T^* \left(\frac{Vol}{\Delta t}\right)_P^{n+1}, \tag{9}$$

$$\gamma_P^* = \left(\frac{\partial \rho}{\partial T}\right)_p^* \left(\frac{Vol}{\Delta t}\right)_P^{n+1}. \tag{10}$$

Equation (8) contains two unknowns, namely, $\overline{p'}$ and $\overline{T'}$, so another equation is required to calculate the global pressure and temperature corrections. This equation is obtained from the finite-difference form of the energy equation which reads

$$M_P^{n+1}T_P^{n+1} - M_P^n T_P^n + \dot{q}_c^{n+1} = \frac{1}{C_p}\left[p_P^{n+1}\left(\frac{Vol}{\Delta t}\right)_P^{n+1} - p_P^n\left(\frac{Vol}{\Delta t}\right)_P^n\right] + S_T^{n+1}, \tag{11}$$

where T and p stand for the temperature and pressure; C_p stands for the specific heat at constant pressure; $\dot{q}_c$ accounts for the total heat flux out of the control volume boundaries by convection and diffusion; and S_T is the source term associated with dissipation, chemical reaction, etc.

Assuming that

$$T_P^{n+1} = T_P^* + \overline{T'}, \tag{12}$$

$$p_P^{n+1} = p_P^* + \overline{p'}, \tag{13}$$

$$M_P^{n+1} = M_P^* + \beta_P^*\overline{p'} + \gamma_P^*\overline{T'} \tag{14}$$

(where the last equation comes from (7) and (3)), substituting (12)–(14) into (11) neglecting nonlinear terms in perturbations, and adding the resulting equation over the whole computational domain we obtain

$$\overline{p'} = \sum_F \left\{M_P^* T_P^* - M_P^n T_P^n + \dot{q}_c^{n+1} - \frac{1}{C_p}\left[p_P^*\left(\frac{Vol}{\Delta t}\right)^{n+1} - p_P^n\left(\frac{Vol}{\Delta t}\right)^n\right] - S_T^{n+1}\right\} \Bigg/ \sum_F \left\{\frac{1}{C_p}\left(\frac{Vol}{\Delta t}\right)^{n+1} - \beta_P^* T_P^*\right\}, \tag{15}$$

which (with (8)) allows to calculate the global pressure and temperature corrections. This use of an explicit formula for $\overline{p'}$ is a significant difference between the present study and the work of Gosman and Watkins [3]. They employed old values for the mass terms in (11). As a result they do not get the cancellation of terms which lead to the explicit form given by (15).

The Local Pressure Correction

To calculate the local pressure correction, and thus, to bring in balance the local continuity equation, we use the finite-difference form of the continuity equation and a staggered grid for the calculation of the flow velocities [1, 2]. Designing the velocities at the cell boundaries of the volume centered at P with the indices of the grid point at which they point, we have, for example,

$$\dot{m}_w^{n+1} = \rho_w^{n+1} u_P^{n+1} a_w = \rho_w^* a_w [u_P^* + D_w(p'_W - p'_P)], \tag{16}$$

where a_w is the area of west boundary, p' stands for the local pressure correction and the asterisk stands for guessed values. Also

$$D_w = \frac{\partial u_P}{\partial (p_W - p_P)} \approx \frac{a_w}{A_P^{n+1}}, \tag{17}$$

where A_P^{n+1} is obtained from the axial momentum equation in finite-difference form:

$$A_P^{n+1} u_P^{n+1} = \sum_c A_c^{n+1} u_C^{n+1} + S_u^{n+1} + A_P^n u_P^n, \tag{18}$$

where $\sum_c$ stands for summation along the u-velocity cell.

Analogous expressions can be obtained for $\dot{m}_e^{n+1}$, $\dot{m}_n^{n+1}$, and $\dot{m}_s^{n+1}$, which after substitution into (4), and taking (14) into account, we obtain

$$M_P^* - M_P^n + \gamma_P^* \overline{T'} + m_c^* + \beta_P^* (\overline{p'} + p'_P) + \sum_c \rho_c a_c D_c (p'_C - p'_P) = 0, \tag{19}$$

which can be identified as the finite-difference form of a Poisson-type equation for the local pressure correction p', which can be written as

$$A_P p'_P = \sum_c A_c p'_C + S_P, \tag{20}$$

where

$$A_c = (\rho a D)_c, \tag{21}$$

$$A_P = \sum_c A_c + \beta_P^*, \tag{22}$$

$$\dot{m}_w^* = \rho_w^* u_P^* a_w, \tag{23}$$

$$S_P = M_P^* - M_P^n - \beta_P^* \overline{p'} - \gamma_P^* \overline{T'} - \dot{m}_c^*. \tag{24}$$

The source term S_P represents the local imbalance of the continuity equation. When $S_P = 0$, the solution of (20) is $p'_P = 0$, because this quantity has a zero normal gradient at the boundaries where the normal velocity is prescribed.

At this point, we remark that the pressure correction is done in two ways; in the first, a global balance of energy and mass is assumed, and in the second, the local continuity equation is employed. However, the temperature is corrected only globally to assure a global balance of energy. The calculation of the global temperature and pressure corrections is only valid when the next iteration within the time step is close enough to the previous iteration to assure the validity of the Taylor series expansion and that the product of perturbations in the energy equation can be neglected.

Comparison with Other Methods

In this section we compare this method with that of Gosman and Watkins [3], and with experimental results obtained with a laser Doppler velocimeter in a four-stroke, axisymmetric piston–cylinder configuration [4]. In this comparison, we have used the same grid, time-step, initial conditions and boundary conditions for both methods. The configuration under study consists of a piston, driven by an electric motor at 31.25 rpm, which slides over a transparent cylinder. Details of the geometry, initial conditions and boundary conditions have been reported by Ramos [4] and are not repeated here. Figures 2 and 3 show the mean axial velocities at three locations inside the cylinder at 240 and 480 crank-angle degrees. A nondimensional coordinate transformation was used to change the moving boundary value problem into a fixed one, and the figures are shown in this transformed space [4]. In these figures the velocity is positive towards the right of the refeence lines and goes to zero at the wall.

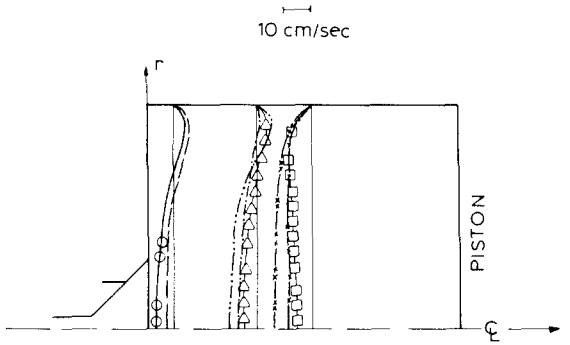

FIG. 2. Mean axial velocity profiles at 240°: (1) Axial location at 0.635 cm: O, experiments; —, present method; – – –, Gosman and Watkins' method. (2) Axial location at 2.54 cm: △, experiment; – · –, present method; – · · –, Gosman and Watkins' method. (3) Axial location at 5.08 cm: □, experiments; – × –, present method; – ×× –, Gosman and Watkins' method.

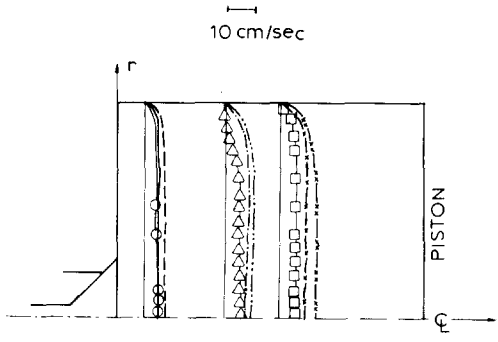

FIG. 3. Mean axial velocity profiles at 480° (for legend see Fig. 2).

Figure 2 shows that our predictions are in better agreement than those obtained using the Gosman and Watkins' method [3] in the compression stroke. Their method shows a bigger recirculation zone, underpredicts the negative velocity at 0.635 cm and overpredicts it at 2.54 and 5.08 cm. This overprediction is substantial at the axial location at 5.08 cm, while the positive velocities at 2.54 cm are in much better agreement than those obtained with the present method. In Fig. 3 the piston, which is moving towards the right of the figure, drives the almost undirectional flow. Both methods overpredict the axial velocity in the power stroke, and show the same flow trends, but Gosman and Watkins' method predicts much higher axial velocities. There are also some savings in computer time. For example, to get the same convergence at 240 and 480 crank-angle degrees our method needed 30 and 21 interactions, while that of Gosman and Watkins method required 41 and 39, respectively. Our method tends to converge faster, except for the first 50 crank-angle degrees after top dead center of the intake. This can only be explained by the fact that the boundary condition for the axial velocity during the intake stroke is proportional to the root square of the pressure difference between the cylinder and atmosphere; however, if this boundary condition is specified as constant, our method tends to converge faster for the entire cycle, i.e., the 720 crank-angle degrees. Globally speaking, at 31.25 rpm and a compression ratio of 7 our method is 1.47 times faster. In the power stroke only, the method is about 1.63 times faster than of Gosman and Watkins[1]; however, for much simpler configurations the two methods have not been compared to each other.

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[1] The same convergence criterion was used for both methods. This criterion states that to advance the time the mass residue in any cell has to be less than 10^{-4} the instantaneous mass flow swept by the piston.